WINDOW BASED PROTOTYPE FILTER DESIGN FOR HIGHLY OVERSAMPLED FILTER BANKS IN AUDIO APPLICATIONS

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ABSTRACT

This paper describes a window based method for designing near perfect-reconstruction prototype filters for highly oversampled, complex modulated filter banks. The design method extends some well-known simple methods for critically sampled filter banks to the oversampled case and to the case of different length analysis and synthesis filters. This design method is simple and effective for designing a large range of filter bank configurations. The design method is particularly useful in developing audio applications using oversampled filter banks where the target system's requirements are highly variable. The simplicity and flexibility of the design method means that this one method can be used to generate multiple prototype filters as the application requirements change.

Index Terms— FIR digital filters, channel bank filters, hearing aids, audio systems

1. INTRODUCTION

Oversampled filter banks have found use in a variety of applications in recent years. In particular, they have found commercial applications in low-power audio signal processing for devices such as hearing aids [1]. Other researchers have also highlighted their potential in audio processing for applications such as acoustic echo cancelation, dynamic range compression and noise reduction. For audio processing devices such as hearing aids, highly oversampled filter banks offer a compromise between aliasing reduction in each subband and achieving ultra low delay through the filter bank, e.g. less than 10 ms with 16 kHz sampling.

Solutions for designing perfect-reconstruction (PR) oversampled filter banks have been presented in the literature [2]. However, the PR condition is routinely violated in audio processing applications. The nature of subband audio processing including subband gain application and subband adaptive filters means that each subband signal is modified between analysis and synthesis such that the PR condition no longer holds. Thus, it is of interest to explore near perfect-reconstruction (NPR) oversampled filter banks. Relaxing the PR condition allows the filter bank designer more flexibility in their application. In the context of audio applications, the amount of distortion allowable in the NPR filter bank will vary with the specific audio processing system. For high-fidelity systems, a dynamic range of 90 dB or significantly above is desirable. For systems that have a higher noise floor due to issues such as microphone noise, a range of only 60 dB may be acceptable. These diverse requirements further emphasize the need for a flexible design strategy that can accommodate a wide variety of filter bank configurations.

This paper presents a simplified method for designing prototype filters for a highly oversampled NPR filter bank using window-based FIR filter designs. This approach will extend similar approaches used in conventional critically sampled NPR filter banks. The goal of this design method is to develop a simple and flexible method that can be applied for a wide variety of oversampled filter bank configurations.

The rest of this paper is organized as follows. First, we review the formulation of an oversampled filter bank including the equations which will be useful in the design method to be presented. Second, we provide a brief summary of other methods in the literature and identify how this new method is different. Finally, we present the new design method and analyze its performance under varying design parameters.

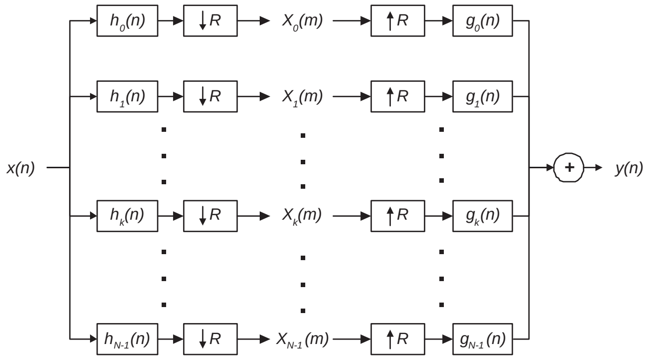

Fig. 1. Block Diagram of an Oversampled Filter Bank

2. OVERSAMPLED FILTER BANKS

This section will briefly review the basic structure of complex modulated oversampled filter banks in order to set a framework for the prototype filter design problem. A block diagram view of an oversampled filter bank is shown in Figure 1. The input signal, $x(n)$, is filtered by N channel filters denoted $h_k(n)$ for $k = 0, 1, \ldots, N-1$. The resulting filtered channel signals are downsampled by a factor of R to create the oversampled subband channel signals $X_k(m)$, where m is a subband time index. The oversampling ratio is then defined as $OS = N/R$, and for highly oversampled filter banks we will consider $OS \geq 2$. The synthesis portion of the filter bank involves upsampling the subband signals by the decimation factor, R, filtering with the synthesis filters (denoted $g_k(n)$) and then summation of the filtered signals.

As usual for these systems, we define the analysis and synthesis channel filters by complex modulation of separate low-pass FIR prototype filters denoted $h(n)$ for analysis and $g(n)$ for synthesis. The channel filters for analysis and synthesis are then described by (1) and (2) respectively. The frequency offset, k_0, allows for creating generalized DFT filter banks including so-called even ($k_0 = 0$) or odd ($k_0 = 0.5$) stacking filter banks [3]. The distinction is not particularly relevant to the prototype filter design method. However, the ability to select different band arrangements is useful in many audio processing applications such as hearing aids [1]. Beyond the band arrangement, the filter bank configuration can be characterized by the number of bands, N, the subband decimation rate, R, the analysis filter length, L_a and the synthesis filter length L_s.

$$h_k(n) = h(n)e^{j\frac{2\pi}{N}(k+k_0)n} \tag{1}$$

$$g_k(n) = g(n)e^{j\frac{2\pi}{N}(k+k_0)n} \tag{2}$$

It is also convenient to use the z-domain view of the filter bank. We use standard notation where a capitalized quantity is used to represent the z-transformed version of a signal or filter, e.g., $H_k(z)$ is the z-transform of $h_k(n)$. Using the z-domain view, the output of the filter bank is given by (3), where $W_R^r = e^{-j\frac{2\pi}{R}r}$.

$$Y(z) = \frac{1}{R}\sum_{r=0}^{R-1}\left[\sum_{k=0}^{N-1} H_k\left(zW_R^r\right)G_k(z)\right] X\left(zW_R^r\right) \tag{3}$$

The portion of (3) for $r = 0$ is the desired output of the filter bank and the terms for $r = 1, \ldots, R-1$ are the aliasing components of the input signal. In a PR filter bank, $Y(z)$ is simply a delayed version of $X(z)$, but for the NPR case we relax this condition such that the $r = 1, \ldots, R-1$ terms must simply be "close" to zero and the $r = 0$ term must simply be "close" to an ideal delay, where the degree of closeness is dependent on the particular application's distortion requirements. These components of (3) can be used to assess the performance of the NPR filter bank. We will denote the $r = 0$ component as $T_d(z)$ using (4). The remaining terms can be summed to form a measure of the aliasing power distortion [4, 5]. We will denote this function as $T_a(z)$ using (5). The functions $T_d(z)$ and $T_a(z)$ will be used to evaluate the performance of the prototype filter design method.

$$T_d(z) = \frac{1}{R}\sum_{k=0}^{N-1} H_k(z)G_k(z) \tag{4}$$

$$T_a(z) = \sqrt{\sum_{r=1}^{R-1}\left|\frac{1}{R}\sum_{k=0}^{N-1} H_k\left(zW_R^r\right)G_k(z)\right|^2} \tag{5}$$

3. EXISTING PROTOTYPE FILTER DESIGN METHODS

Most recent design methods for NPR oversampled filter banks have focused on careful formulation of an optimization problem involving a minimum stopband energy objective function and constraints which enforce the NPR condition [6, 7]. Other popular methods involve iterative least squares solutions to an objective function which is a weighted sum of the amplitude distortion and stopband attenuation [8]. Many of the oversampled filter bank design methods restrict themselves to the case where $h(n)$ and $g(n)$ are based on the same underlying prototype and are of equal length. In addition, these methods are designed without high oversampling ratios in mind and some, such as the iterative methods, are ill-conditioned for higher oversampling.

In the past, researchers had developed very simple but effective methods for critically sampled NPR filter banks. The design method to be presented in this paper draws on some of these examples, in particular those from [9] and [10]. In these examples, the prototype filter design problem is restricted to classes of filters such as those designed using the Parks-McLellan algorithm or those designed using a Kaiser window based approach. In particular, the approach of [10] provides a simple design method where the stopband attenuation is controlled through selection of the Kaiser window and filter length, whereas the overall filter bank distortion is minimized by adjusting the cutoff frequency of the designed filter.

The design method presented in this paper will extend existing critically sampled window based design methods to oversampled filter banks. Window based prototype filter designs for oversampled filter banks are also presented in [11]. However, this research uses nonlinear optimization methods to solve for the optimal prototype filter. In our method, we will return to the objective function and design method presented in [10]. This method is simple and effective across a wide variety of filter bank configurations. Modifications are also made to the method so that the synthesis filter can be a decimated version of the analysis filter.

4. WINDOW BASED DESIGN METHOD

Our new method takes the window-based FIR filter design methods presented in [10] for critically sampled filter banks and extends it in two key ways. First, we apply the general method to oversampled filter banks. Second, we implement a design where one filter is a decimated version of the other. The objective function for this design method is formulated by (6).

$$\phi = \max_{0\leq\omega<\frac{\pi}{N}} \left| |H_0(e^{j\omega})G_0(e^{j\omega})|^2 + |H_1(e^{j\omega})G_1(e^{j\omega})|^2 - 1 \right| \tag{6}$$

The goal is to minimize ϕ , which is a measure of the peak amplitude distortion for the first two analysis/synthesis filter pairs, over the frequency region of the first subband. Because the amplitude distortion is periodic with period π/N, we can perform the optimization only over the frequency region of the first band.

Drawing on the previous research examples, we employ classic window-based FIR filter design methods to design the prototype filters $h(n)$ and $g(n)$ [10, 11]. The use of a window-based design requires that the designer select a filter length and window which will allow them to reach the stopband attenuation desired. Using window based FIR filter designs, we define an arbitrary prototype filter, $p(n)$, as the product of an L-point finite length window, $w(n)$, and the infinite length ideal low-pass filter for a certain cut-off frequency (ω_c):

$$p(n) = w(n)p_0(n) \quad n = 0, \ldots, L-1 \tag{7}$$

$$p_0(n) = \frac{\sin\left(\omega_c(n - \frac{L-1}{2})\right)}{\pi\left(n - \frac{L-1}{2}\right)} \tag{8}$$

Based on the empirical evidence from other researchers, we expect that ϕ will be a convex function of the filter cut-off frequency in the neighborhood of the ideal cut-off frequency, π/N [9, 10]. Under this assumption, it is straightforward to optimize the window cut-off

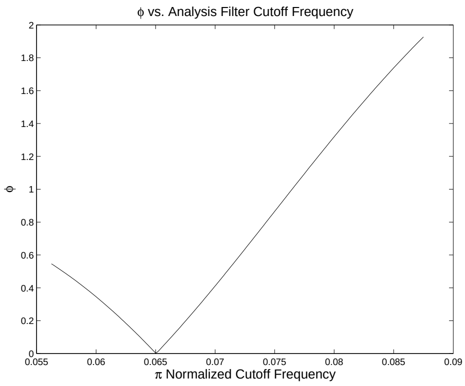

Fig. 2. Filter Design Objective Function - $L_a = 64$, $L_s = 32$, $N = 16$, $R = 4$, Hamming Window

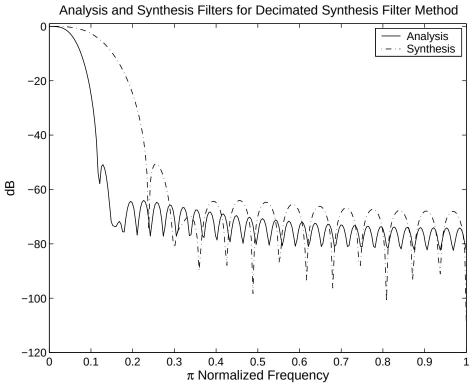

Fig. 3. Prototype Filters - $L_a = 64$, $L_s = 32$, $N = 16$, $R = 4$, Hamming window

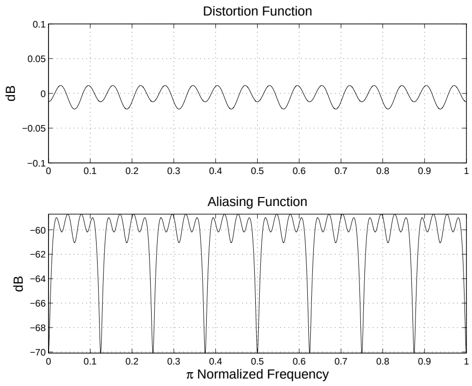

Fig. 4. Distortion and Aliasing Functions - $L_a = 64$, $L_s = 32$, $N = 16$, $R = 4$, Hamming window

frequency using any number of simple techniques such as an iterative search or an exhaustive search over a suitably dense number of frequencies.

In order to address the ability to design separate analysis and synthesis filters, we reduce the two prototype filter design problem back to a one prototype filter design problem by defining the synthesis filter to be a decimated version of the analysis filter in the manner suggested by [1]. For this method, we define an integer decimation factor $D = L_a/L_s$ and the synthesis prototype filter to be $g(n) = h(Dn)$. Since $h(n)$ is generally band limited to π/N, this decimation only introduces aliasing in the synthesis filter at the level of the stopband attenuation of the analysis filter provided that $D \leq OS$. Having a decimated synthesis filter reduces the delay of the filter bank which is important for real-time audio processing applications. Using this method, the optimization problem is reduced again to a simple function of the analysis filter's cutoff frequency and D becomes an additional design parameter for the filter bank configuration.

5. PROTOTYPE FILTER DESIGN ANALYSIS AND RESULTS

This section will analyze some results of using the filter design method described in Section 4. The goal is to verify that the method works as designed and to investigate the tradeoffs involved when using it for various filter bank configurations. The primary means of evaluation will be the transfer function and aliasing distortion functions given by (4) and (5).

5.1. Objective Function and Performance Analysis

One of the empirical results for critically sampled filter banks which used similar filter design methods is that the objective function is convex in the area around the ideal filter cut-off frequency [9, 10]. This is illustrated for one example of an oversampled filter bank in Figure 2, where the objective function is plotted versus the cutoff frequency used in the prototype filter design. This example uses $L_a = 64$, $L_s = 32$, $N = 16$ and $R = 4$ with a Hamming window function. Given this result, it is reasonable to assume that the design method will always find a local optimum if it confines the search to an area around these ideal cutoff frequencies.

The resulting analysis and synthesis filters for this example are shown in Figure 3. The transfer function distortion and aliasing functions, i.e. $T_d(z)$ and $T_a(z)$, are shown in Figure 4. From these results, we see that this design method produces an excellent NPR filter bank response and aliasing distortion at the same order of magnitude as the stopband attenuation of the prototype filters, as would be expected.

5.2. Flexibility Analysis

For a wide range of audio applications, it is desirable to assess the design method's flexibility under variations of the filter bank parameters. We will focus on two parameters - the filter length and the number of bands. The filter length directly affects the system performance and contributes to the filter bank group delay. The number of bands controls the frequency resolution provided by the filter bank. Other filter bank parameters such as the oversampling ratio have a direct effect of the filter bank performance, but are not directly used in the filter design method. Since the filter design method produces minimal levels of filter bank distortion through its objective function, the primary performance metric for this analysis will be the average aliasing power, which is derived from the aliasing distortion function in (5).

To study the effect of filter length, we will re-use the $N = 16$, $R = 4$ filter bank design from previous sections for this analysis, but we will now vary the analysis and synthesis filter lengths. We will also now use a Kaiser window to demonstrate the design method's

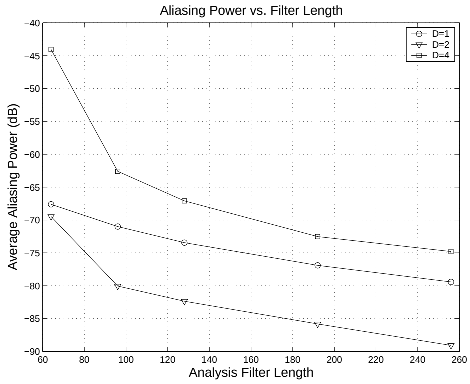

Fig. 5. Average Aliasing Power vs. Analysis Filter Length using a Kaiser window and $N = 16$, $R = 4$

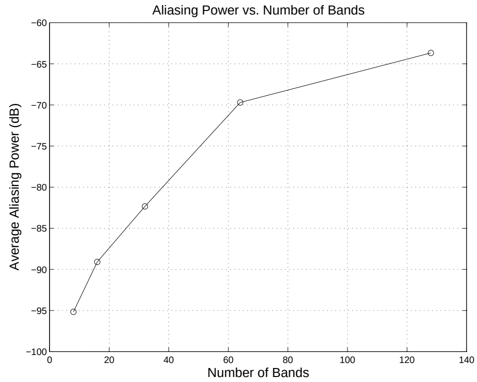

Fig. 6. Average Aliasing Power vs. Number of Bands using a Kaiser window and $L_a = 256$, $L_s = 128$, $N = 8, 16, 32, 64, 128$ and $R = N/4$

use with a different windowing function. Figure 5 shows the average aliasing power for a variety of analysis filter lengths using varying ratios of $D = L_a/L_s$. The results show that the method scales well for different filter lengths, with an expected correlation between reduced aliasing power and longer filters. The results for a $D = 1$ filter bank match what would be designed for an equivalent critically sampled filter bank with identical analysis and synthesis prototypes. An interesting result is that the $D = 1$ designs perform marginally poorer than the $D = 2$ designs for this average aliasing performance metric. One would intuitively expect that a longer synthesis filter will reduce the aliasing power. However, in the $D = 1$ case we have only one single prototype filter which must be power complementary to itself in order to minimize the linear distortions. This condition requires that the prototype cutoff frequency be higher than in the $D = 2$ case, thereby allowing a slightly increased amount of aliasing.

Figure 6 shows the average aliasing power as a function of the number of bands in the filter bank. The analysis and synthesis filters were fixed to $L_a = 128$ and $L_s = 64$. The oversampling ratio (N/R) was also fixed to 4, so that R was scaled accordingly with N. The results are as expected - given a constant filter length we get improved performance with fewer bands since the cutoff frequency requirements of the prototype filter are not as stringent. More importantly however, this shows that the filter design method is well behaved across a range of filter bank sizes.

6. CONCLUSION

This paper has presented a window based prototype filter design method for near-perfect reconstruction oversampled filter banks. This method was based on previous similar work using critically sampled filter banks. The method and the resulting oversampled filter banks are targeted at low delay audio applications where the processing requirements can vary during the system design. A simple, flexible method like this reduces the complexity and difficulty in designing prototype filters for a wide range of filter bank configurations. Future work on this method includes applying it using other FIR filter methods as well as exploring alternate methods for different analysis and synthesis filters.

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