Simulating Power Supplies with SPICE
Agenda

- Why simulating power supplies?
- Average modeling techniques
- The PWM switch concept, CCM
- The PWM switch concept, DCM
- The voltage-mode model at work
- Current-mode modeling
- The current-mode model at work
- Power factor correction
- Switching models
- EMI filtering
- Conclusion
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Why Simulate Switch Mode Power Supplies?

- Simulation helps feeling how the product behaves before breadboard
- Experiment What If? at any level. Power libraries do not blow!
- Easily shows impact of parameter variations: ESR, Load etc.
- Draw Bode plots without using costly equipments
- Avoid trials and errors: compensate the loop on the PC first!
- Use SPICE to assess current amplitudes, voltage stresses etc.
- Go to the lab. and check if the assumptions were valid.

SPICE does **NOT** replace the breadboard!
Why Average Simulations?

- An average model is made of equations that are continuous in time
- The switching component has disappeared, leading to:
  - a simpler ac analysis of the power supply
  - the study of the stability margins in various conditions
  - the assessment of the ESRs contributions in the loop stability
  - a flashing simulation time!
Why *Switching* Simulations?

- An switching model is like breadboarding on the PC
- The switching component is back in place, leading to:
  - the analysis of current and voltage stresses
  - the study of leakage and stray elements impacts
  - the analysis of the input current signature – EMI
  - a longer simulation time…

![Switching approach diagram]
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Average Modeling, the SSA

- State-Space Averaging (SSA)
- Introduced by Slobodan Ćuk in the 80’s
- Long and painful process
- Fails to predict sub-harmonic oscillations

\[ \frac{dx_1}{dt} = -\frac{1}{L} x_2 + \frac{1}{L} u \]

\[ \frac{dx_2}{dt} = \frac{1}{C_{out}} x_1 - \frac{1}{R_{load}.C_{out}} x_2 \]

Apply smoothing process  \rightarrow  Linearize  😞
Average Modeling, the PWM Switch

- The PWM Switch
- Introduced by Vatché Vorpérian in the mid-80’s
- Easy to derive and fully invariant
- No auto-toggling mode models
- Can predict sub-harmonic oscillations in CCM
- DCM model in current-mode was never published!

Diagram:
- PWM switch
- Convention of notation:
  - $I_a(t)$, $I_c(t)$
  - $V_{ap(t)}$, $V_{cp(t)}$
  - $V_{in}$, $V_{out}$
The PWM Switch Concept

- Identify the guilty network: the transistor and the diode
- Average their voltage and current waveforms: large-signal model
- Linearize the equations around a dc point: small-signal model

Diode + transistor = guilty for non-linearity!
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The PWM Switch Concept

- The transistor is a highly non-linear device:
  - Replace the transistor with its small-signal model
  - Solve a system of linear equations

Replace \( Q_1 \) by its small-signal model

Remember the **bipolars**
Ebers-Moll model...
The PWM Switch Concept

- The PWM switch model works in all two switch converters:
  - Rotate the model to match the switch and diode connections
  - Solve a system of linear equations

**Buck-boost**

**Boost**

**Buck**
The PWM Switch Concept

- The keyword with average modeling: waveforms averaging

\[ \langle I_a (t) \rangle_{T_{sw}} = I_a = \frac{1}{T_{sw}} \int_0^{T_{sw}} I_a (t) \, dt = d \langle I_c (t) \rangle_{T_{sw}} = dI_c \]

\[ \langle V_{cp} (t) \rangle_{T_{sw}} = V_{cp} = \frac{1}{T_{sw}} \int_0^{T_{sw}} V_{cp} (t) \, dt = d \langle V_{ap} (t) \rangle_{T_{sw}} = dV_{ap} \]
The PWM Switch Concept

- The obtained set of equations is that of a transformer
  - A CCM two-switch DC-DC can be modeled like a 1:D transformer!

Large-signal (non-linear) model
The PWM Switch Concept

- SPICE only deals with linear equations
- It first computes a bias point then it linearizes the network

Always verify the dc operating point!

- No equations, result appears in a second!
- Make sure the bias point is correct…
The PWM Switch Concept

- We have a set of non-linear equations: can’t derive transfer functions!
- We need a small-signal model: linearize the equations by hand
  - two options: perturbation or partial derivatives...

### Perturbation

\[
\begin{align*}
I_a &= dI_c \\
I_a &= I_{a0} + \hat{i}_a \\
I_c &= I_{c0} + \hat{i}_c \\
d &= d_0 + \hat{d} \\
I_{a0} + \hat{i}_a &= (d_0 + \hat{d})(I_{c0} + \hat{i}_c)
\end{align*}
\]

\[
\begin{align*}
I_{a0} &= d_0 I_{c0} \\
V_{cp0} &= d_0 V_{ap0}
\end{align*}
\]

### Partial derivatives

\[
\begin{align*}
\dot{I}_a &= \frac{\partial I_a}{\partial d} \dot{d} \\
\dot{V}_{cp} &= \frac{\partial V_{cp}}{\partial d} \dot{d}
\end{align*}
\]

\[
\begin{align*}
\dot{\dot{I}}_a &= d_0 \dot{i}_c + \hat{d} I_{c0} \\
\dot{\dot{V}}_{cp} &= d_0 \dot{v}_{ap} + \hat{d} V_{ap0}
\end{align*}
\]
The PWM Switch Concept

- Put the small-signal sources in the large-signal model
- You obtain the small-signal model of the CCM PWM switch

You can now analytically find the dc bias and the ac response!
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The PWM Switch in DCM

- The original model could not be auto-toggling
- A new DCM-CCM model has been derived

3rd event linked to DCM

\[ I_a = \frac{I_{peak} d_1}{2} \]
\[ I_c = \frac{I_{peak} (d_1 + d_2)}{2} \]
\[ I_c = I_a \frac{(d_1 + d_2)}{d_1} \]
The PWM Switch in DCM

- By clamping the $d_2$ equation, the circuit toggles between the modes.

$$\text{N} = \frac{d_1}{d_1 + d_2}$$

Clamp $d_2$:
- $d_2$ CCM = $1 - d_1$
- $d_2$ DCM = $1 - d_1 - d_3$

$d_2 < 1 - d_1$
model is in DCM!

$$d_2 = \frac{2I_c L - V_{ac} d_1^2 T_{sw}}{V_{ac} d_1 T_{sw}} = \frac{2LF_{sw}}{d_1} \frac{I_c}{V_{ac}} [d_1]$$

Model input
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The PWM Switch in DCM

- In voltage-mode, the duty-cycle is built with a ramp generator.
- The transition occurs when the error voltage crosses the ramp.

\[
\begin{align*}
V_{\text{err}}(t) &= V_{\text{peak}} \frac{t_{\text{on}}(t)}{T_{\text{sw}}} = V_{\text{peak}} d(t) \\
d(t) &= \frac{V_{\text{err}}(t)}{V_{\text{peak}}} \quad \text{and} \\
\frac{d}{d\left(\frac{V_{\text{err}}(t)}{V_{\text{peak}}}\right)} &= \frac{1}{V_{\text{peak}}} = K_{\text{PWM}}
\end{align*}
\]
The Voltage-Mode Model at Work

- Let us compensate a buck converter operated in CCM and DCM
  1. Run an open-loop Bode plot at full load, lowest input
  2. Identify the excess/deficiency of gain at the selected cross over
  3. Place a double zero at $f_0$, a pole at the ESR zero and a pole at $F_{sw}/2$
  4. Check final loop gain and run a transient load test

Parameters

- $R_{upper}=38k$
- $f_c=7k$
- $G_{fc}=-15$
- $G=10^{^-G_{fc}/20}$
- $\pi=3.14159$
- $f_z1=650$
- $f_z2=650$
- $f_{p1}=7k$
- $f_{p2}=50k$
- $C_3=1/(2\pi f_z1 R_{upper})$
- $R_3=1/(2\pi f_z2 C_3)$
- $C_1=1/(2\pi f_z2 R_2)$
- $C_2=1/(2\pi (f_{p1}) R_2)$

Automated compensation

- $a=f_c^4+f_c^2 f_z1^2+2+f_c^2 f_z2^2+f_z1^2 f_z2^2$
- $c=f_p2^2 f_{p1}^2+f_c^2 f_p2^2+f_c^2 f_{p1}^2+f_c^4$
- $R_2=\sqrt{c/a}G^*f_c R_3/f_{p1}$
The Voltage-Mode Model at Work

- The Bode plot reveals a gain loss of -15 dB at 7 kHz
- The compensator provides a +15 dB gain increase plus phase boost

The final loop gain shows a comfortable phase margin
The transient response at both input levels shows a stable signal

\[ |H(s)| \]
\[ \arg |H(s)| \]
\[ \text{Arg } H(7 \text{ kHz}) = -121^\circ \]
\[ f_c = 7 \text{ kHz} \]
\[ P_m = 80^\circ \]

\[ |T(s)| \]
\[ \text{Arg } T(s) \]

\[ V_{out}(t) \]
\[ I_{out} = 200 \text{ mA to } 4 \text{ A in } 10 \mu s \]
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Current-Mode Operation

- In voltage-mode, the error signal directly controls the duty cycle
- In current mode, the error voltage sets the inductor peak current
- To derive a model, observe the current signals and average them!

\[ I_c(t)R_i = V_c(t) - d(t)T_{sw}S_e - \frac{S_f d'(t)T_{sw}}{2} \]

\[ I_c = \frac{V_c}{R_i} d \frac{T_{sw}S_e}{R_i} - V_{cp} (1 - d) \frac{T_{sw}}{2L} \]
Current-Mode Operation

- Do the same for DCM signals
- Match the previous structure to build a CCM/DCM model

\[ I_{\text{c}} = \frac{V_c - d_1 T_{\text{sw}} S_e}{R_i} - \alpha d_2 T_{\text{sw}} S_f \]

\[ I_{\mu} = \frac{d_1 T_{\text{sw}} S_e}{R_i} + d_2 T_{\text{sw}} \frac{V_{cp}}{L} \left(1 - \frac{d_1 + d_2}{2}\right) \]
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The Current-Mode Model at Work

- To study a converter, we can write down the equations
- Or use a SPICE simulation to get the Bode plot in a second
- Take the example of a current-mode flyback converter

\[
|H(f)| = 20 \log_{10} \frac{G_0}{\sqrt{1 + \left(\frac{f}{f_{z1}}\right)^2} \sqrt{1 + \left(\frac{f}{f_{z2}}\right)^2} \sqrt{1 + \left(\frac{f}{f_{z3}}\right)^2} \sqrt{1 + \left(\frac{f}{f_{p1}}\right)^2} \sqrt{1 - \left(\frac{f}{f_n}\right)^2} + \left(\frac{f}{f_{nQ_p}}\right)^2}}
\]

\[
\arg H(f) = \tan^{-1}\left(\frac{f}{f_{z1}}\right) - \tan^{-1}\left(\frac{f}{f_{z2}}\right) + \tan^{-1}\left(\frac{f}{f_{z3}}\right) - \tan^{-1}\left(\frac{f}{f_{p1}}\right) - \tan^{-1}\left(\frac{f}{f_{nQ_p}}\right) \left(\frac{1}{1 - \left(\frac{f}{f_n}\right)^2}\right)
\]
Stabilizing a CCM Flyback Converter

- Capture a SPICE schematic with an averaged model

- Look for the bias points values: $V_{out} = 19$ V, ok
- $V_{setpoint} < 1$ V, enough margin on current sense
Stabilizing a CCM Flyback Converter

- Capture a SPICE schematic with an averaged model

```plaintext
parameters
Vout=19
Ibridge=250u
Rlower=2.5/Ibridge
Rupper=(Vout-2.5)/Ibridge
Lp=350u
Se=20k
fc=1k
pm=60
Gfc=-22
pfc=-71
from
Bode
G=10^(-Gfc/20)
boost=pm-(pfc)-90
pi=3.14159
K=tan((boost/2+45)*pi/180)
Fzero=fc/k
Fpole=k*fc
Rpullup=20k
RLED=CTR*Rpullup/G
Czero=1/(2*pi*Fzero*Rupper)
Cpole=1/(2*pi*Fpole*Rpullup)
CTR=1.5
Pole=6k
```

```plaintext
from
Bode
G=10^(-Gfc/20)
boost=pm-(pfc)-90
pi=3.14159
K=tan((boost/2+45)*pi/180)
Fzero=fc/k
Fpole=k*fc
Rpullup=20k
RLED=CTR*Rpullup/G
Czero=1/(2*pi*Fzero*Rupper)
Cpole=1/(2*pi*Fpole*Rpullup)
CTR=1.5
Pole=6k
```
Stabilizing a CCM Flyback Converter

- Capture a SPICE schematic with an averaged model

Gain at 1 kHz
-22 dB

Phase at 1 kHz
-71°

|H(s)|

Sub harmonic poles

Inject ramp compensation

Inject ramp compensation

Gain at 1 kHz
-22 dB

Phase at 1 kHz
-71°
Stabilizing a CCM Flyback Converter

- The easiest way to damp the poles:
  - Calculate the equivalent quality coefficient at $F_{sw}/2$
  - Calculate the external ramp to make $Q$ less than 1

$$Q = \frac{1}{\pi \left( D \left( \frac{S_e}{S_n} - \frac{1}{2} - D \right) \right)} = \frac{1}{3.14 \times (0.5 - 0.46)} = 8$$

$$S_e = \frac{S_n}{D'} \left( \frac{1}{\pi} - 0.5 + D \right) = \frac{V_{in} R_i}{L_p D'} \left( \frac{1}{\pi} - 0.5 + D \right) = \frac{90 \times 0.25}{320 \times (1 - 0.46)} \left( \frac{1}{3.14} - 0.5 + 0.46 \right) = 36 \text{ kV/s}$$

2.3 $V_{pp}$

$R_{ramp}$ 18 kΩ

$S_{ramp} = \frac{2.3}{15u} = 153 \text{ kV/s}$

$R_{current} = \frac{M_r S_n R_{ramp}}{S_{ramp}} = \frac{0.51 \times 70k \times 18k}{153k} = 4.1 \text{ kΩ}$

$M_r = \frac{S_e}{S_n} = \frac{36k}{70k} = 51\%$ On-time slope $\frac{V_{in} R_i}{L_p}$
Stabilizing a CCM Flyback Converter

- Boost the gain by +22 dB, boost the phase at $f_c$.

Cross over

1 kHz

GM 20 dB

$|T(s)|$

arg$T(s)$

Margin at 1 kHz

60°
Stabilizing a CCM Flyback Converter

- Test the response at both input levels, 90 and 265 Vrms
- Sweep ESR values and check margins again

![Graph showing output voltage $V_{out}(t)$ over time with high and low line indicators and a peak of 112 mV.](image-url)
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Power Factor Correction

- The bulk capacitor connects to a low-impedance source
- At the bulk capacitor refueling, a narrow peak current flows
- This peak conveys a large harmonic content

The bulk capacitor connects to a low-impedance source. At the bulk capacitor refueling, a narrow peak current flows. This peak conveys a large harmonic content.
Power Factor Correction

- A pre-converter is installed as a front-end section
- The pre-converter draws a sinusoidal current
- The energy is stored and released in/by the bulk capacitor
Power Factor Correction

- One of the most popular techniques uses Borderline mode.
- The MC33262 operates in peak current mode control.
- The NCP1606 also operates in constant-on time.
Power Factor Correction

- The core is always reset from cycle to the other

- The average inductor current is half the inductor peak current value
Power Factor Correction

- A 150 W BCM PFC average example with the MC33262

![Diagram with circuit components and parameters]

- **Current-mode borderline model**

- **Parameter Values:**
  - \( V_{\text{rms}} = 100 \)
  - \( P_{\text{out}} = 150 \)
  - \( V_{\text{out}} = 400 \)
  - \( R_i = 0.22 \)
  - \( L = 850 \mu \text{H} \)
Power Factor Correction

- Average models can also work in transient conditions

- High line
  - \( V_{in} = 230 \, V_{ac} \)
  - THD = 10%
  - \( I_{in}(t) \)

- Constant on-time
  - \( V_{out\,peak} = 406 \, V \)
  - \( V_{out\,valley} = 398 \, V \)

- Low line
  - \( V_{in} = 100 \, V_{ac} \)
  - THD = 2%
  - \( I_{in}(t) \)
Power Factor Correction

- Use the model to boost the phase at the cross over point

\[ f_c = 5 \text{ Hz} \]

\[ P_m = 61^\circ \]

Zero added

No zero added
Power Factor Correction

- The zero improves the overshoot but degrades the THD...

![Graph showing voltage and current waveforms with added zero vs. no zero, THD calculations, and voltage and current values.](image-url)
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Switching Models, the Breadboard on PC

Turn your PC into a virtual breadboard

Ohmic losses

Primary inductance

Leakage inductance

Pulse generator

Vinput 300V

Capacitor parasitics

D1 1N5818

Resr 100m

Rload10

Vout

Cout 100uF

Lp 1mH

Leak 10uH

Rs 100m

1:N
Switching Models, the Breadboard on PC

Wire your device as you would do in the lab.
Simulations (Really) Work!!

- Assess the average, rms currents in your circuit
- Check if enough margins exist on your semiconductors
Simulations (Really) Work!!

- With accurate models, the simulation results are excellent.
- You can then vary the parasitic terms and see their impact.

![Graph showing simulated vs. measured data with 30mV/div and 2ms/div scales.](image)

simulated  measured
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EMI Filtering on a DC-DC

- DC-DC are highly EMI polluting systems
- A filter has to be installed to avoid noise in the source

Need to limit the ac current < 15 mA peak
EMI Filtering on a DC-DC

- Use SPICE to extract the current signature
- Run Fourier analysis to look at the spectrum

\[ I_{\text{avg}} = 1.7 \, \text{A} \]
\[ I_{\text{rms}} = 2.7 \, \text{A} \]

Input current signature

\[ I_{\text{peak}} = 2.45 \, \text{A} \]

FFT results

\[ I_{\text{ripple}} < 15 \, \text{mA} \]
\[ A_{\text{filter}} < 15 \text{m} \times 6 \]
\[ f_0 < \sqrt{0.006 \times F_{\text{sw}}} < 7.7 \, \text{kHz} \]

Position the cutoff frequency of the LC filter
EMI Filtering on a DC-DC

- A LC configuration offers the best efficiency
- As any LC network, it is subject to resonances

$$L = 100 \, \mu H$$

$$C = \frac{1}{4\pi^2 f_0^2 L} = 5.2 \, \mu F$$

Check impedance peaking

$$||Z_{outFILTER}||_{\text{max}} = \frac{Z_0^2}{R_1} \sqrt{1 + \left(\frac{R_1}{Z_0}\right)^2}$$

$$7.7 \, \text{kHz}$$
EMI Filtering on a DC-DC

- The incremental input resistance of a DC-DC in negative
- A LC filter loaded by a negative resistance can oscillate!

The incremental input resistance of a DC-DC in negative

- A LC filter loaded by a negative resistance can oscillate!

The incremental input resistance of a DC-DC in negative

- A LC filter loaded by a negative resistance can oscillate!

The incremental input resistance of a DC-DC in negative

- A LC filter loaded by a negative resistance can oscillate!

The incremental input resistance of a DC-DC in negative

- A LC filter loaded by a negative resistance can oscillate!

The incremental input resistance of a DC-DC in negative

- A LC filter loaded by a negative resistance can oscillate!
EMI Filtering on a DC-DC

- If the resonance is too peaky, problems can arise
EMI Filtering on a DC-DC

- A resistor is damping the LC filter by creating losses
- A dc-block capacitor is installed to limit dissipation

\[
R_{\text{damp}} = - Z_{\text{in SMPS}} \frac{L + CR_1 R_2 - \frac{R_1}{\omega_0}}{2 Z_{\text{in SMPS}} CR_1 - \frac{Z_{\text{in SMPS}}}{\omega_0} + L + CR_2 R_1 - \frac{R_1}{\omega_0}}
\]
EMI Filtering on a DC-DC

- The right resistor prevents the overlaps between curves
EMI Filtering on a DC-DC

- A final check shows a noise amplitude under control

\[ y \text{ (pk-pk)} = 22.8 \text{mA amperes} \text{ between 3.91m and 3.92m seconds} \]
A Book on Power Supply Design

- To learn more about power supplies and simulations…

886 pages, 8 chapters

- Learn DC-DC converters theory
- Understand average modeling
- Feedback and loop control
- Design examples of DC-DC and AC-DC
- Power Factor Correction
- Chapters on flyback and forward converters
- Supplied CDROM with working examples

I already have ideas for the next edition!!
Conclusion

- SPICE can be seen as a design companion
- It shields us from going through complex equations
- Simulation time is short and PC helps to run tests
- Use SPICE before going to the bench: NO trial and error!
- Once the simulation is stable, build the prototype
- Simulations and laboratory debug: the success recipe!
For More Information

• View the extensive portfolio of power management products from ON Semiconductor at www.onsemi.com

• View reference designs, design notes, and other material supporting the design of highly efficient power supplies at www.onsemi.com/powersupplies