AND9105/D

Biquad Filters in ON Semiconductor Pre-configured Digital Hybrids

ON Semiconductor®

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APPLICATION NOTE

Introduction

Pre-configured products offered by ON Semiconductor offer great flexibility in adjusting input/output characteristics as well as shaping of the system's frequency response. The input/output characteristics of the system can be adjusted by algorithms such as WDRC compression, AGC-O, peak clipping and low level expansion (squelch) blocks. The system's frequency response can be controlled by adjusting individual channel's gain and the crossover frequencies between channels. Also, the system's frequency response can be controlled using biquadratic (biquad) filters.

ON Semiconductor pre-configured products often include eight biquad filters: four pre-emphasis and four post-emphasis. The pre-emphasis and post-emphasis filters are located before and after the filter bank respectively. These filters can be configured through ARKonline to be parametric filters with assignable frequency, amplitude, and Q. Certain filters can also be assigned as low-cut or high-cut filters with adjustable corner frequency and filter order. Please refer to ARKonline for further details on filter configuration. Additionally, each filter can be configured as "generic" biquad filters where the coefficients are directly exposed and can be assigned at a later time. Generic biquads offer the most flexibility and can accommodate various filter types.

This application note is intended to provide a brief tutorial on how to construct generic biquad filter values which can be directly imported into the Interactive Data Sheet (IDS). Once imported and enabled, the filter will impact the overall frequency response of the product.

NOTE: It is also possible to construct filter coefficients automatically using the Filter Designer Plus (FDP) application which is included in the ARKbase™ distribution package. The FDP allows for a variety of different filter types to be constructed via the graphical user interface and also supports automatic transferring of coefficients to the IDS. Further information on the FDP can be found in the ARK User's Guide**.**

Biquad Filter Structure

All generic biquad filters implemented on ON Semiconductor pre-configured products are IIR (Infinite Impulse Response) in Direct Form 1 structure (see Figure 1).

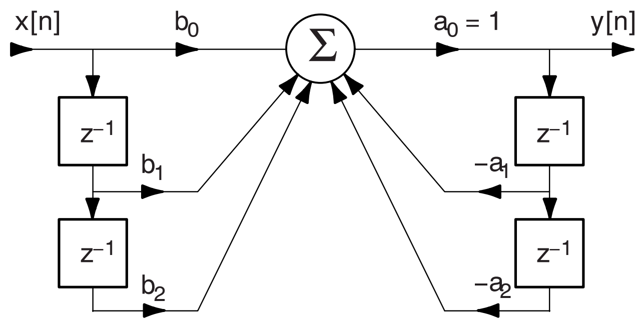

Figure 1. Direct Form 1 IIR Structure Block Diagram

The transfer function of each of the biquad filter can be expressed as follows:

$$H(z) = \frac{b_0 + b_1 \times z^{-1} + b_2 \times z^{-2}}{1 + a_1 \times z^{-1} + a_2 \times z^{-2}}$$

NOTE: The coefficient a_0 is hard−wired to always be 1 (see Figure 1). The coefficients are each 16 bits in length and include one sign bit, one bit to the left of the decimal point, and 14 bits to the right of the decimal point, thus having the format 1s1i14f. Before quantization, the floating−point coefficients must be in the range $-2.0 \le x < 2.0$ and quantized with the function:

$$round(\, x \times 2^{14})$$

All multiplications and additions are done with a single multiplier-accumulator (MAC), and therefore the following rules should be considered for the biquad filter to function properly:

- All coefficients {b0, b1, b2, a1, a2} must be in the range [-2,+2).
- The calculated filter's output signal level should not exceed the full scale representation in the filter. For example, if the input signal to the biquad is -10 dBFS (10 dB below Full Scale), therefore the biquad has only 10 dB of headroom before it starts overflowing.

NOTE: When transferring coefficients to the IDS, the underlying code in the product components automatically checks all of the filters in the system for stability (i.e., the poles have to be within the unit circle) before updating the graphs on the screen when the filter is enabled or programming the coefficients into the hybrid. If the IDS receives an exception from the underlying stability checking code, it will automatically disable the biquad being modified and display a warning message. When the filter is made stable again, it can be re−enabled.

Digital System Limitations

When designing and implementing an IIR filter, it is important to consider the limitations of any DSP system. The limitations described here can cause the IIR filter to become noisy, distorting or even unstable. In that case, it may be necessary to adjust some of the filter's parameters so it will perform as expected.

When the signal is converted from analog to digital form, the precision is limited by the number of bits available. The signals are not only subject to limited precision, filter coefficients follow the same rule – they are also subject to limited precision of digital systems. The limited precision of all digital systems leads to quantization errors.

Those errors due to limited precision are nonlinear and signal dependent. The nonlinearity can eventually lead to instability, particularly with IIR filters.

There is no method that can eliminate quantization errors. However those errors can be minimized by keeping values large so that the maximum number of bits is used to represent them. However, there is a limit to how large the numbers can be.

There is also another potential source of error in DSP systems, particularly in IIR filters. It is possible for a result of IIR filter processing to exceed the maximum allowable size. When that happens, the hardware may saturate or overflow. To avoid the overflow or saturation, the input signal and/or filter coefficients should be scaled down.

Digital IIR Filter Design

The following examples illustrate different types of IIR filter designs. Each filter is first designed as an analog filter and the filter's characteristics (i.e., order, corner frequency, passband gain) are translated into an analog transfer function. Poles and zeros of the analog transfer function are matched to the poles and zeros of the digital filter's transfer function using bilinear transformation and then quantized to integer values which can be entered in the generic biquad section of the IDS. Those examples require some knowledge of complex math and *s* and *z* operators used in analog and digital transfer functions.

- **Example 1:** 1^{st} order low pass filter with 4 kHz corner frequency and 0 dB gain:

$$H(s) = \frac{\omega_c}{s + \omega_c}$$

where $\omega_c = 2\pi f_c$, f_c is −3 dB corner frequency in Hz, $\pi = 3.1415935$

The transfer function has pole s_{p1} at $-\omega_c$ and zero s_{z1} at ∞. The bilinear transformation is used to convert the analog transfer function to a digital transfer function. The pole and the zero of the digital transfer function are matched to the pole and zero of the analog transfer function according to the following formula:

$$BT(s) = \frac{1 + T/2 \times s}{1 - T/2 \times s}$$

where T is the stamping period, $T = 1/f_s$, and f_s is the sampling frequency.

The corner frequency of the analog filter $f_c = 4000$ Hz, therefore $\omega_c = 25132$. The transfer function of this analog low pass filter is:

$H(s) = \frac{25132}{s + 25132}$, the pole $s_{p1} = -25132$, the zero $s_{z1} = \infty$

Using the bilinear transformation, sampling frequency $f_s = 32000$ Hz:

$$z_{p1} = \frac{1 + (-25132 / 64000)}{1 - (-25132 / 64000)}$$

$$z_{p1} = 0.4360604,\ z_{z1} = -1$$

The transfer function of the first order digital filter is:

$$H(z) = G \times \frac{z - z_{z1}}{z - z_{p1}}$$

Gain parameter G has to be calculated so the digital filter has unity gain at DC:

$$H(z) = G \times \frac{1 - z_{z1}}{1 - z_{p1}} = 1,\ thus\ G = \frac{1 - z_{p1}}{2}$$

$$BT(0) = 1$$

G = 0.2819698

The transfer function of the digital filter:

$$H(z) = 0.2819698 \times \frac{z + 1}{z - 0.4360604} =$$

$$= \frac{0.2819698 \times z + 0.2819698}{z - 0.4360604}$$

The filter's coefficients have to be quantized to 1s1i14f format before being entered in the IDS. The conversion is done by multiplying the real coefficients of the transfer function by 2^{14} and rounding the results to the nearest integer value. The rounded integers can then be entered into one of the generic biquad sections in the IDS for the selected product. The resulting values are given in the table below:

Coefficient	Real Value	Quantized Value
b0	0.2819698	4620
b1	0.2819698	4620
b2	0	0
a0	1	16384
a1	-0.4360604	-7144
a2	0	0

- **Example 2:** 1st order high pass filter with 500 Hz corner frequency and 0 dB gain

The transfer function of the 1st order analog high pass filter with 0 dB gain:

$$H(s) = \frac{s}{s + \omega_c}$$

where $\omega_c = 2\pi f_c$, f_c is −3 dB corner frequency in Hz, $\pi = 3.1415935$

The transfer function has pole s_{p1} at $-\omega_c$ and zero s_{z1} at 0. The bilinear transformation is used to convert the analog transfer function to a digital transfer function. The pole and the zero of the digital transfer function are matched to the pole and zero of the analog transfer function according to the following formula:

$$z = \frac{1 + T/2 \times s}{1 - T/2 \times s}$$

where T is stamping period, $T = 1/f_s$, and f_s is the sampling frequency.

The corner frequency of the analog high pass filter $f_c = 500$ Hz, therefore $\omega_c = 3141$. The transfer function of this analog low pass filter is:

$$H(s) = \frac{s}{s + 3141}$$

$$s_{p1} = -3141,\ s_{z1} = 0$$

Using the bilinear transformation, sampling frequency $f_s = 32000$ Hz:

$$z_{p1} = \frac{1 + (-3141 / 64000)}{1 - (-3141 / 64000)}$$

$$z_{p1} = 0.9064189,\ z_{z1} = 1$$

The transfer function of the first order digital filter is:

$$H(z) = G \times \frac{z - z_{z1}}{z - z_{p1}}$$

Gain parameter G has to be calculated so the digital filter has unity gain at infinite frequency:

$$H(z) = G \times \frac{-1 - z_{z1}}{-1 - z_{p1}} = 1,\ thus\ G = \frac{1 + z_{p1}}{2}$$

$$BT(\infty) = -1$$

G = 0.9532094

$$H(z) = 0.9532094 \times \frac{z - 1}{z - 0.9064189} =$$

$$= \frac{0.9532094 \times z - 0.9532094}{z - 0.9064189}$$

The filter’s coefficients have to be quantized as in Example 1. The resulting values are given in the table below:

Coefficient	Real Value	Quantized Value
b0	0.9532094	15617
b1	−0.9532094	−15617
b2	0	0
a0	1	16384
a1	−0.9064189	−14851
a2	0	0

- **Example 3:** 2^{nd} order low pass filter with 3000 Hz corner frequency and 0 dB gain

The transfer function of a 2nd order analog low pass filter with 0 dB gain:

$$H(s) = \frac{\omega_c^2}{s^2 + 2 \times \zeta \times \omega_c \times s + \omega_c^2}$$

ζ = damping ratio (0 to 1)

$\omega_c = 2\pi f_c$, f_c = corner frequency

The transfer function has two poles located at:

$$s_{p1,p2} = -\zeta \times \omega_c \pm j \times \omega_c \times \sqrt{1 - \zeta^2}$$

and two zeros $s_{z1} = s_{z2}$ at ∞

The corner frequency of the 2^{nd} order analog filter f_c = 3000 Hz, therefore ω_c = 18850. For the 2^{nd} order filter to have −3 dB at the corner frequency the damping ratio should be set to:

$$\zeta = \frac{\sqrt{2}}{2} = 0.7071068$$

The transfer function of the analog low pass filter:

$$H(s) = \frac{3.553 \times 10^8}{s^2 + 2.6657 \times 10^4 \times s + 3.553 \times 10^8}$$

the poles $s_{p1,p2} = -13329 \pm j \times 13329$,

the zeros $s_{z1} = s_{z2} = \infty$

Using the bilinear transformation, sampling frequency f_s = 32000 Hz:

$$z_{p1} = \frac{1 + \frac{(-13329 + j \times 13329)}{64000}}{1 - \frac{(-13329 + j \times 13329)}{64000}}$$

$$z_{p1} = 0.6075147 + j\,0.2770771$$

$$z_{p2} = \frac{1 + \frac{(-13329 - j \times 13329)}{64000}}{1 - \frac{(-13329 - j \times 13329)}{64000}}$$

$$z_{p2} = 0.6075147 - j\,0.2770771$$

$$z_{z1} = z_{z2} = -1$$

The second order digital filter transfer function:

$$H(z) = G \times \frac{(z - z_{z1}) \times (z - z_{z2})}{(z - z_{p1}) \times (z - z_{p2})}$$

$$H(z) = G \times \frac{z^2 + 2 \times z + 1}{z^2 - 1.2150293 \times z + 0.4458458}$$

Gain parameter G has to be calculated so the digital filter has unity gain at DC:

$$H(z) = G \times \frac{(1 - z_{z1}) \times (1 - z_{z2})}{(1 - z_{p1}) \times (1 - z_{p2})} = 1$$

$$BT(0) = 1$$

thus $G = \frac{(1 - z_{p1}) \times (1 - z_{p2})}{4}$

G = 0.0577041

The transfer function of the digital filter:

$$H(z) = \frac{0.0577041\,z^2 + 0.1154082\,z + 0.0577041}{z^2 - 1.2150293 \times z + 0.4458458}$$

The filter's coefficients have to be quantized as in Example 1. The resulting values are given in the table below:

Coefficient	Real Value	Quantized Value
b0	0.0577041	945
b1	0.1154082	1891
b2	0.0577041	945
a0	1	16384
a1	−1.2150293	−19907
a2	0.4458458	7305

- **Example 4:** 2^{nd} order high pass filter with 1000 Hz corner frequency and 0 dB gain

The transfer function of a 2nd order analog high pass filter with 0 dB gain:

$$H(s) = \frac{s^2}{s^2 + 2 \times \zeta \times \omega_c \times s + \omega_c{}^2}$$

ζ = damping ratio (0 to 1)

$\omega_c = 2\pi f_c$, f_c = corner frequency

The transfer function has two poles located at

$$s_{p1,p2} = -\zeta \times \omega_c \pm j \times \omega_c \times \sqrt{1-\zeta^2}$$

and two zeros $s_{z1} = s_{z2}$ at 0.

The corner frequency of the 2^{nd} order analog filter f_c = 1000 Hz, therefore ω_c = 6283.

For the 2^{nd} order filter to have −3 dB at the corner frequency the damping ratio should be set to:

$$\zeta = \frac{\sqrt{2}}{2} = 0.7071068$$

The transfer function of the analog low pass filter:

$$H(s) = \frac{s^2}{s^2 + 8.8858 \times 10^3 \times s + 3.9478 \times 10^7}$$

the poles $s_{p1,p2} = -4443 \pm j \times 4443,$

the zeros $s_{z1} = s_{z2} = 0$

Using the bilinear transformation, sampling frequency f_s = 32000 Hz:

$$z_{p1} = \frac{1 + \frac{(-4443 + j \times 4443)}{64000}}{1 - \frac{(-4443 + j \times 4443)}{64000}}$$

$$z_{p1} = 0.86232501 + j \times 0.1208905$$

$$z_{p2} = \frac{1 + \frac{(-4443 - j \times 4443)}{64000}}{1 - \frac{(-4443 - j \times 4443)}{64000}}$$

$$z_{p2} = 0.86232501 - j \times 0.1208905$$

$$z_{z1} = z_{z2} = 1$$

The second order digital filter transfer function:

$$H(z) = G \times \frac{(z - z_{z1}) \times (z - z_{z2})}{(z - z_{p1}) \times (z - z_{p2})}$$

$$H(z) = G \times \frac{z^2 - 2 \times z + 1}{z^2 - 1.7246502 \times z + 0.7582191}$$

Gain parameter G has to be calculated so the digital filter has unity gain at infinity:

$$H(z) = G \times \frac{(-1 - z_{z1}) \times (-1 - z_{z2})}{(-1 - z_{p1}) \times (-1 - z_{p2})} = 1$$

$$BT(\infty) = -1$$

thus $G = \frac{(-1 - z_{p1}) \times (-1 - z_{p2})}{4}$

G = 0.8707173

The transfer function of the digital filter:

$$H(z) = \frac{0.8707173 \times z^2 - 1.7414346 \times z + 0.8707173}{z^2 - 1.7246502 \times z + 0.7582191}$$

The filter's coefficients have to be quantized as in Example 1. The resulting values are given in the table below:

Coefficient	Real Value	Quantized Value
b0	0.8707173	14266
b1	−1.7414346	−28532
b2	0.8707173	14266
a0	1	16384
a1	−1.7246502	−28257
a2	0.7582191	12423

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